Key concepts and equations to know. Make notecards for each circled equation (and any that you don't know), but know all equations and concepts.

100 Key Math Concepts for the ACT

Number Properties

1. **UNDEFINED**

On the ACT, *undefined* almost always means **division by zero**. The expression $\frac{a}{bc}$ is undefined if either b or c equals 0.

2. **REAL/IMAGINARY**

A real number is a number that has a **location on the number line**. On the ACT, imaginary numbers are numbers that involve the square root of a negative number. $\sqrt{-4}$ is an imaginary number.

3. **INTEGER/NONINTEGER**

Integers are **whole numbers**; they include negative whole numbers and zero.

4. **RATIONAL/IRRATIONAL**

A **rational number** is a number that can be expressed as a **ratio of two integers**. **Irrational numbers** are real numbers—they have locations on the number line—they just **can't be expressed precisely as a fraction or decimal**. For the purposes of the ACT, the most important **irrational numbers** are $\sqrt{2}$, $\sqrt{3}$, and π.

5. **ADDING/SUBTRACTING SIGNED NUMBERS**

To **add a positive and a negative**, first ignore the signs and find the positive difference between the number parts. Then attach the sign of the original number with the larger number part. For example, to add 23 and –34, first we ignore the minus sign and find the positive difference between 23 and 34—that's 11. Then we attach the sign of the number with the larger number part—in this case it's the minus sign from the –34. So, $23 + (-34) = -11$.

Make **subtraction** situations simpler by turning them into addition. For example, think of $-17 - (-21)$ as $-17 + (+21)$.

To **add or subtract a string of positives and negatives**, first turn everything into addition. Then combine the positives and negatives so that the string is reduced to the sum of a single positive number and a single negative number.

6. **MULTIPLYING/DIVIDING SIGNED NUMBERS**

To multiply and/or divide positives and negatives, treat the number parts as usual and **attach a negative sign if there were originally an odd number of negatives**. To multiply –2, –3, and –5, first multiply the number parts: $2 \times 3 \times 5 = 30$. Then go back and note that there were *three*—an *odd* number—negatives, so the product is negative: $(-2) \times (-3) \times (-5) = -30$.

7. PEMDAS

When performing multiple operations, remember **PEMDAS**, which means **Parentheses** first, then **Exponents**, then **Multiplication** and **Division** (left to right), then **Addition** and **Subtraction** (left to right).

In the expression $9 - 2 \times (5 - 3)^2 + 6 \div 3$, begin with the parentheses: $(5 - 3) = 2$. Then do the exponent: $2^2 = 4$. Now the expression is: $9 - 2 \times 4 + 6 \div 3$. Next do the multiplication and division to get $9 - 8 + 2$, which equals 3.

8. ABSOLUTE VALUE

Treat absolute value signs a lot like **parentheses**. Do what's inside them first and then take the absolute value of the result. Don't take the absolute value of each piece between the bars before calculating. In order to calculate $|(-12) + 5 - (-4)| - |5 + (-10)|$, first do what's inside the bars to get: $|-3| - |-5|$, which is $3 - 5$, or -2.

9. COUNTING CONSECUTIVE INTEGERS

To count consecutive integers**, subtract the smallest from the largest and add 1**. To count the integers from 13 through 31, subtract: $31 - 13 = 18$. Then add 1: $18 + 1 = 19$.

Divisibility

10. FACTOR/MULTIPLE

The **factors** of integer *n* are the positive integers that divide into *n* with no remainder. The **multiples** of *n* are the integers that *n* divides into with no remainder. 6 is a factor of 12, and 24 is a multiple of 12. 12 is both a factor and a multiple of itself.

11. PRIME FACTORIZATION

A **prime number** is a positive integer that has exactly two positive integer factors: 1 and the integer itself. The first eight prime numbers are 2, 3, 5, 7, 11, 13, 17, and 19.

To find the prime factorization of an integer, just keep breaking it up into factors until **all the factors are prime**. To find the prime factorization of 36, for example, you could begin by breaking it into 4×9:

$36 = 4 \times 9 = 2 \times 2 \times 3 \times 3$

12. RELATIVE PRIMES

To determine whether two integers are relative primes, break them both down to their prime factorizations. For example: $35 = 5 \times 7$, and $54 = 2 \times 3 \times 3 \times 3$. They have **no prime factors in common**, so 35 and 54 are relative primes.

13. COMMON MULTIPLE

You can always get a common multiple of two numbers by **multiplying** them, but, unless the two numbers are relative primes, the product will not be the *least* common multiple. For example, to find a common multiple for 12 and 15, you could just multiply: $12 \times 15 = 180$.

14. LEAST COMMON MULTIPLE (LCM)

To find the least common multiple, check out the **multiples of the larger number** until you find one that's **also a multiple of the smaller.** To find the LCM of 12 and 15, begin by taking the multiples of 15: 15 is not divisible by 12; 30's not; nor is 45. But the next multiple of 15, 60, *is* divisible by 12, so it's the LCM.

15. GREATEST COMMON FACTOR (GCF)

To find the greatest common factor, break down both numbers into their prime factorizations and take **all the prime factors they have in common**. $36 = 2 \times 2 \times 3 \times 3$, and $48 = 2 \times 2 \times 2 \times 2 \times 3$. What they have in common is two 2s and one 3, so the GCF is $= 2 \times 2 \times 3 = 12$.

2×2 = e   2+2 = e
2×3 = e   2+1 = o
3×3 = o   1+1 = e

16. EVEN/ODD

To predict whether a sum, difference, or product will be even or odd, just **take simple numbers like 1 and 2 and see what happens**. There are rules—"odd times even is even," for example—but there's no need to memorize them. What happens with one set of numbers generally happens with all similar sets.

17. MULTIPLES OF 2 AND 4

An integer is divisible by 2 if the **last digit is even**. An integer is divisible by 4 if the **last two digits form a multiple of 4**. The last digit of 562 is 2, which is even, so 562 is a multiple of 2. The last two digits make 62, which is *not* divisible by 4, so 562 is not a multiple of 4.

18. MULTIPLES OF 3 AND 9

An integer is divisible by 3 if the **sum of its digits is divisible by 3**. An integer is divisible by 9 if the **sum of its digits is divisible by 9**. The sum of the digits in 957 is 21, which is divisible by 3 but not by 9, so 957 is divisible by 3 but not 9.

19. MULTIPLES OF 5 AND 10

An integer is divisible by 5 if the **last digit is 5 or 0**. An integer is divisible by 10 if the **last digit is 0**. The last digit of 665 is 5, so 665 is a multiple 5 but *not* a multiple of 10.

20. REMAINDERS

The remainder is the **whole number left over after division**. 487 is 2 more than 485, which is a multiple of 5, so when 487 is divided by 5, the remainder will be 2.

Fractions and Decimals

21. REDUCING FRACTIONS

To reduce a fraction to lowest terms, **factor out and cancel** all factors the numerator and denominator have in common.

$$\frac{28}{36} = \frac{4 \times 7}{4 \times 9} = \frac{7}{9}$$

22. ADDING/SUBTRACTING FRACTIONS

To add or subtract fractions, first find a **common denominator**, and then add or subtract the numerators.

$$\frac{2}{15} + \frac{3}{10} = \frac{4}{30} + \frac{9}{30} = \frac{4+9}{30} = \frac{13}{30}$$

23. MULTIPLYING FRACTIONS

To multiply fractions, **multiply** the numerators and **multiply** the denominators.

$$\frac{5}{7} \times \frac{3}{4} = \frac{5 \times 3}{7 \times 4} = \frac{15}{28}$$

24. DIVIDING FRACTIONS

To divide fractions, **invert** the second one and **multiply**.

$$\frac{1}{2} \div \frac{3}{5} = \frac{1}{2} \times \frac{5}{3} = \frac{1 \times 5}{2 \times 3} = \frac{5}{6}$$

25. CONVERTING A MIXED NUMBER TO AN IMPROPER FRACTION

To convert a mixed number to an improper fraction, **multiply** the whole number part by the denominator, then **add** the numerator. The result is the new numerator (over the same denominator). To convert $7\frac{1}{3}$, first multiply 7 by 3, then add 1, to get the new numerator of 22. Put that over the same denominator, 3, to get $\frac{22}{3}$.

26. CONVERTING AN IMPROPER FRACTION TO A MIXED NUMBER

To convert an improper fraction to a mixed number, **divide** the denominator into the numerator to get a **whole number quotient with a remainder**. The quotient becomes the whole number part of the mixed number, and the remainder becomes the new numerator—with the same denominator. For example, to convert $\frac{108}{5}$, first divide 5 into 108, which yields 21 with a remainder of 3. Therefore, $\frac{108}{5} = 21\frac{3}{5}$.

27. RECIPROCAL

To find the reciprocal of a fraction, **switch** the numerator and the denominator. The reciprocal of $\frac{3}{7}$ is $\frac{7}{3}$. The reciprocal of 5 is $\frac{1}{5}$. The product of reciprocals is 1.

use calculator

28. COMPARING FRACTIONS

One way to compare fractions is to re-express them with a **common denominator**.

$\frac{3}{4} = \frac{21}{28}$ and $\frac{5}{7} = \frac{20}{28}$. $\frac{21}{28}$ is greater than $\frac{20}{28}$, so $\frac{3}{4}$ is greater than $\frac{5}{7}$.

Another way to compare fractions is to convert them both to **decimals**. $\frac{3}{4}$ converts to .75, and $\frac{5}{7}$ converts to approximately .714.

29. CONVERTING FRACTIONS TO DECIMALS

To convert a fraction to a decimal, **divide the bottom into the top**. To convert $\frac{5}{8}$, divide 8 into 5, yielding .625.

30. REPEATING DECIMAL

To find a particular digit in a repeating decimal, note the **number of digits in the cluster that repeats**. If there are 2 digits in that cluster, then every 2nd digit is the same. If there are 3 digits in that cluster, then every 3rd digit is the same. And so on. For example, the decimal equivalent of $\frac{1}{27}$ is .037037037..., which is best written $.\overline{037}$.

There are 3 digits in the repeating cluster, so every 3rd digit is the same: 7. To find the 50th digit, look for the multiple of 3 just less than 50—that's 48. The 48th digit is 7, and with the 49th digit the pattern repeats with 0. The 50th digit is 3.

31. IDENTIFYING THE PARTS AND THE WHOLE

The key to solving most story problems involving fractions and percents is to identify the part and the whole. Usually you'll find the **part** associated with the verb ***is/are*** and the **whole** associated with the word ***of***. In the sentence, "Half of the boys are blonds," the whole is the boys ("*of* the boys"), and the part is the blonds ("*are* blonds").

Percents

32. PERCENT FORMULA

Whether you need to find the part, the whole, or the percent, use the same formula:

Part = Percent × Whole

Example: What is 12% of 25?
Setup: Part = .12 × 25

Example: 15 is 3% of what number?
Setup: 15 = .03 × Whole

Example: 45 is what percent of 9?
Setup: 45 = Percent × 9

33. PERCENT INCREASE AND DECREASE

To increase a number by a percent, **add the percent to 100%**, convert to a decimal, and multiply. To increase 40 by 25%, add 25% to 100%, convert 125% to 1.25, and multiply by 40. 1.25 × 40 = 50.

34. FINDING THE ORIGINAL WHOLE

To find the **original whole before a percent increase or decrease**, set up an equation. Think of a 15% increase over x as $1.15x$.

Example: After a 5% increase, the population was 59,346. What was the population *before* the increase?

Setup: $1.05x = 59{,}346$

35. COMBINED PERCENT INCREASE AND DECREASE

To determine the combined effect of multiple percents increase and/or decrease, **start with 100 and see what happens**.

Example: A price went up 10% one year, and the new price went up 20% the next year. What was the combined percent increase?

Setup: First year: 100 + (10% of 100) = 110. Second year: 110 + (20% of 110) = 132. That's a combined 32% increase.

Ratios, Proportions, and Rates

36. SETTING UP A RATIO

To find a ratio, put the number associated with the word ***of* on top** and the quantity associated with the word ***to* on the bottom** and reduce. The ratio of 20 oranges to 12 apples is $\frac{20}{12}$ which reduces to $\frac{5}{3}$.

37. PART-TO-PART AND PART-TO-WHOLE RATIOS

If the parts add up to the whole, a part-to-part ratio can be turned into two part-to-whole ratios by putting **each number in the original ratio over the sum of the numbers**. If the ratio of males to females is 1 to 2, then the males-to-people ratio is

$\frac{1}{1+2} = \frac{1}{3}$ and the females-to-people ratio is

$\frac{2}{1+2} = \frac{2}{3}$. Or, $\frac{2}{3}$ of all the people are female.

38. SOLVING A PROPORTION

To solve a proportion, **cross multiply:**

$$\frac{x}{5} = \frac{3}{4}$$

$$4x = 5 \times 3$$

$$x = \frac{15}{4} = 3.75$$

39. RATE = proportion problem

To solve a rates problem, **use the units** to keep things straight.

Example: If snow is falling at the rate of 1 foot every 4 hours, how many inches of snow will fall in 7 hours?

Setup:

Cross multiply ratios

$$\frac{1 \text{ foot}}{4 \text{ hours}} = \frac{x \text{ inches}}{7 \text{ hours}}$$

$$\frac{12 \text{ inches}}{4 \text{ hours}} = \frac{x \text{ inches}}{7 \text{ hours}}$$

$$4x = 12 \times 7$$

$$x = 21$$

40. AVERAGE RATE

Average rate is *not* simply the average of the rates.

$$\text{Average } A \text{ per } B = \frac{\text{Total } A}{\text{Total } B}$$

$$\text{Average Speed} = \frac{\text{Total distance}}{\text{Total time}}$$

To find the average speed for 120 miles at 40 mph and 120 miles at 60 mph, **don't just average the two speeds**. First figure out the total distance and the total time. The total distance is 120 + 120 = 240 miles. The times are 3 hours for the first leg and 2 hours for the second leg, or 5 hours total. The average speed, then, is $\frac{240}{5} = 48$ miles per hour.

Averages

41. AVERAGE FORMULA

To find the average of a set of numbers, **add them up and divide by the number of numbers**.

$$\text{Average} = \frac{\text{Sum of the terms}}{\text{Number of terms}}$$

To find the average of the five numbers 12, 15, 23, 40, and 40, first add them: 12 + 15 + 23 + 40 + 40 = 130. Then divide the sum by 5: 130 ÷ 5 = 26.

42. AVERAGE OF EVENLY SPACED NUMBERS

To find the average of evenly spaced numbers, just **average the smallest and the largest**. The average of all the integers from 13 through 77 is the same as the average of 13 and 77. $\frac{13+77}{2} = \frac{90}{2} = 45$

43. USING THE AVERAGE TO FIND THE SUM

$$\text{Sum} = (\text{Average}) \times (\text{Number of terms})$$

If the average of ten numbers is 50, then they add up to 10 × 50, or 500.

44. FINDING THE MISSING NUMBER

To find a missing number when you're given the average, **use the sum**. If the average of four numbers is 7, then the sum of those four numbers is 4 × 7, or 28. Suppose that three of the numbers are 3, 5, and 8. These numbers add up to 16 of that 28, which leaves 12 for the fourth number.

Possibilities and Probability

45. COUNTING THE POSSIBILITIES

The fundamental counting principle: if there are ***m* ways** one event can happen and ***n* ways** a second event can happen, then there are **$m \times n$ ways** for the two events to happen. For example, with 5 shirts and 7 pairs of pants to choose from, you can put together $5 \times 7 = 35$ different outfits.

46. PROBABILITY

desired

$$\text{Probability} = \frac{\text{Favorable outcomes}}{\text{Total possible outcomes}}$$

If you have 12 shirts in a drawer and 9 of them are white, the probability of picking a white shirt at random is $\frac{9}{12} = \frac{3}{4}$. This probability can also be expressed as .75 or 75%.

Powers and Roots

47. MULTIPLYING AND DIVIDING POWERS

To multiply powers with the same base, **add the exponents:** $x^3 \cdot x^4 = x^{3+4} = x^7$. To divide powers with the same base, **subtract the exponents**: $y^{13} \div y^8 = y^{13-8} = y^5$.

48. RAISING POWERS TO POWERS

To raise a power to an exponent, **multiply the exponents**. $(x^3)^4 = x^{3 \times 4} = x^{12}$.

49. SIMPLIFYING SQUARE ROOTS

To simplify a square root, **factor out the perfect squares** under the radical, unsquare them and put the result in front. $\sqrt{12} = \sqrt{4 \times 3} = \sqrt{4} \times \sqrt{3} = 2\sqrt{3}$.

50. ADDING AND SUBTRACTING ROOTS

You can add or subtract radical expressions **only if the part under the radicals is the same**.

$2\sqrt{3} + 3\sqrt{3} = 5\sqrt{3}$

51. MULTIPLYING AND DIVIDING ROOTS

The product of square roots is equal to the **square root of the product**:

$\sqrt{3} \times \sqrt{5} = \sqrt{3 \times 5} = \sqrt{15}$. The quotient of square roots is equal to the **square root of the quotient**: $\frac{\sqrt{6}}{\sqrt{3}} = \sqrt{\frac{6}{3}} = \sqrt{2}$.

Algebraic Expressions

52. EVALUATING AN EXPRESSION

To evaluate an algebraic expression, **plug in** the given values for the unknowns and calculate according to PEMDAS. To find the value of $x^2 + 5x - 6$ when $x = -2$, plug in -2 for x:
$(-2)^2 + 5(-2) - 6 = 4 - 10 - 6 = -12$.

53. ADDING AND SUBTRACTING MONOMIALS

To combine like terms, **keep the variable part unchanged while adding or subtracting the coefficients**. $2a + 3a = (2 + 3)a = 5a$

54. ADDING AND SUBTRACTING POLYNOMIALS

To add or subtract polynomials, **combine like terms.**

$(3x^2 + 5x - 7) - (x^2 + 12) =$
$(3x^2 - x^2) + 5x + (-7 - 12) = 2x^2 + 5x - 19$

55. MULTIPLYING MONOMIALS

To multiply monomials, **multiply the coefficients and the variables separately**.
$2a \times 3a = (2 \times 3)(a \times a) = 6a^2$.

56. MULTIPLYING BINOMIALS—FOIL

To multiply binomials, use **FOIL**. To multiply $(x + 3)$ by $(x + 4)$, first multiply the **F**irst terms: $x \times x = x^2$. Next the **O**uter terms: $x \times 4 = 4x$. Then the **I**nner terms: $3 \times x = 3x$. And finally the **L**ast terms: $3 \times 4 = 12$. Then add and combine like terms: $x^2 + 4x + 3x + 12 = x^2 + 7x + 12$.

57. MULTIPLYING OTHER POLYNOMIALS

FOIL works only when you want to multiply two binomials. If you want to multiply polynomials with more than two terms, make sure you **multiply each term in the first polynomial by each term in the second.**

$(x^2 + 3x + 4)(x + 5) =$
$x^2(x + 5) + 3x(x + 5) + 4(x + 5) =$
$x^3 + 5x^2 + 3x^2 + 15x + 4x + 20 =$
$x^3 + 8x^2 + 19x + 20$

Factoring Algebraic Expressions

58. FACTORING OUT A COMMON DIVISOR

A factor common to all terms of a polynomial can be **factored out**. All three terms in the polynomial $3x^3 + 12x^2 - 6x$ contain a factor of $3x$. Pulling out the common factor yields $3x(x^2 + 4x - 2)$.

59. FACTORING THE DIFFERENCE OF SQUARES

One of the test maker's favorite factorables is the **difference of squares**.

$$a^2 - b^2 = (a - b)(a + b)$$

$x^2 - 9$, for example, factors to $(x - 3)(x + 3)$.

60. FACTORING THE SQUARE OF A BINOMIAL

Learn to recognize polynomials that are squares of binomials:

$$a^2 + 2ab + b^2 = (a + b)^2$$

$$a^2 - 2ab + b^2 = (a - b)^2$$

For example, $4x^2 + 12x + 9$ factors to $(2x + 3)^2$, and $n^2 - 10n + 25$ factors to $(n - 5)^2$.

61. FACTORING OTHER POLYNOMIALS—FOIL IN REVERSE

To factor a quadratic expression, **think about what binomials you could use FOIL on to get that quadratic expression**. To factor $x^2 - 5x + 6$, think about what **F**irst terms will produce x^2, what **L**ast terms will produce +6, and what **O**uter and **I**nner terms will produce $-5x$. Common sense—and trial and error—lead you to $(x - 2)(x - 3)$.

62. SIMPLIFYING AN ALGEBRAIC FRACTION

Simplifying an algebraic fraction is a lot like simplifying a numerical fraction. The general idea is to **find factors common to the numerator and denominator and cancel them**. Thus, simplifying an algebraic fraction begins with factoring.

To simplify $\frac{x^2 - x - 12}{x^2 - 9}$ first factor the numerator and denominator: $\frac{x^2 - x - 12}{x^2} - 9 = \frac{(x - 4)(x + 3)}{(x - 3)(x + 3)}$

Canceling $x + 3$ from the numerator and denominator leaves you with $\frac{x - 4}{x - 3}$.

Solving Equations

63. SOLVING A LINEAR EQUATION

To solve an equation, do whatever is necessary to both sides to **isolate the variable**. To solve $5x - 12 = -2x + 9$, first get all the x's on one side by adding $2x$ to both sides: $7x - 12 = 9$. Then add 12 to both sides: $7x = 21$, then divide both sides by 7 to get: $x = 3$.

64. SOLVING "IN TERMS OF"

To solve an equation for one variable **in terms of** another means to **isolate the one variable on one side of the equation**, leaving an expression containing the other variable on the other side. To solve $3x - 10y = -5x + 6y$ for x in terms of y, isolate x:

$$\begin{aligned} 3x - 10y &= -5x + 6y \\ 3x + 5x &= 6y + 10y \\ 8x &= 16y \\ x &= 2y \end{aligned}$$

65. TRANSLATING FROM ENGLISH INTO ALGEBRA

To translate from English into algebra, look for the key words and systematically turn phrases into algebraic expressions and sentences into equations. Be careful about order, especially when subtraction is called for.

Example: The charge for a phone call is r cents for the first 3 minutes and s cents for each minute thereafter. What is the cost, in cents, of a call lasting exactly t minutes? ($t > 3$)

Setup: The charge begins with r, and then something more is added, depending on the length of the call. The amount added is s times the number of minutes past 3 minutes. If the total number of minutes is t, then the number of minutes past 3 is $t - 3$. So the charge is $r + s(t - 3)$.

Intermediate Algebra

66. SOLVING A QUADRATIC EQUATION

To solve a quadratic equation, put it in the $ax^2 + bx + c = 0$ form, **factor** the left side (if you can), and set each factor equal to 0 separately to get the two solutions. To solve $x^2 + 12 = 7x$, first rewrite it as $x^2 - 7x + 12 = 0$. Then factor the left side:

$$(x - 3)(x - 4) = 0$$
$$x - 3 = 0 \text{ or } x - 4 = 0$$
$$x = 3 \text{ or } 4$$

Sometimes the left side might not be obviously factorable. You can always use the **quadratic formula**. Just plug in the coefficients a, b, and c from $ax^2 + bx + c = 0$ into the formula:

$$\boxed{\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

To solve $x^2 + 4x + 2 = 0$, plug $a = 1$, $b = 4$, and $c = 2$ into the formula:

$$x = \frac{-4 \pm \sqrt{4^2 - 4 \times 1 \times 2}}{2 \times 1}$$
$$= \frac{-4 \pm \sqrt{8}}{2} = -2 \pm \sqrt{2}$$

67. SOLVING A SYSTEM OF EQUATIONS

You can solve for two variables only if you have two distinct equations. Two forms of the same equation will not be adequate. **Combine the equations** in such a way that **one of the variables cancels out**. To solve the two equations $4x + 3y = 8$ and $x + y = 3$, multiply both sides of the second equation by -3 to get: $-3x - 3y = -9$. Now add the equations; the $3y$ and the $-3y$ cancel out, leaving: $x = -1$. Plug that back into either one of the original equations and you'll find that $y = 4$.

68. SOLVING AN EQUATION THAT INCLUDES ABSOLUTE VALUE SIGNS

To solve an equation that includes absolute value signs, **think about the two different cases**. For example, to solve the equation $|x - 12| = 3$, think of it as two equations:

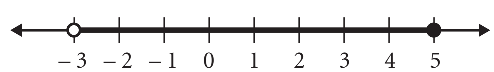
$$x - 12 = 3 \text{ or } x - 12 = -3$$
$$x = 15 \text{ or } 9$$

69. SOLVING AN INEQUALITY

To solve an inequality, do whatever is necessary to both sides to **isolate the variable**. Just remember that when you **multiply or divide both sides by a negative number**, you must **reverse the sign**. To solve $-5x + 7 < -3$, subtract 7 from both sides to get: $-5x < -10$. Now divide both sides by -5, remembering to reverse the sign: $x > 2$.

70. GRAPHING INEQUALITIES

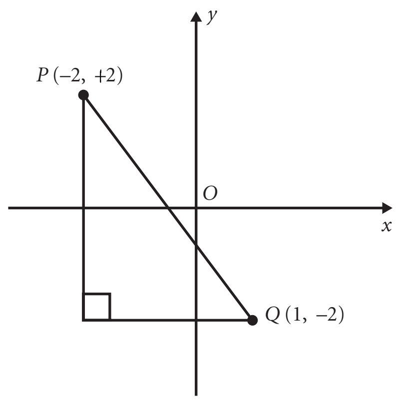
To graph a range of values, use a thick, black line over the number line, and at the end(s) of the range, use a **solid circle** if the point ***is* included** or an **open circle** if the point is ***not* included**. The figure here shows the graph of $-3 < x \le 5$.



Coordinate Geometry

71. FINDING THE DISTANCE BETWEEN TWO POINTS

To find the distance between points, **use the Pythagorean theorem or special right triangles.** The difference between the x's is one leg and the difference between the y's is the other leg.



In the figure above, $\overline{PQ}$ is the hypotenuse of a 3-4-5 triangle, so $\overline{PQ} = 5$.

You can also use the **distance formula**:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

To find the distance between $R(3, 6)$ and $S(5, -2)$:

$$\begin{aligned} d &= \sqrt{(5-3)^2 + (-2-6)^2} \\ &= \sqrt{(2)^2 + (-8)^2} \\ &= \sqrt{68} = 2\sqrt{17} \end{aligned}$$

72. USING TWO POINTS TO FIND THE SLOPE

In mathematics, the slope of a line is often called m.

$$\text{Slope} = m = \frac{\text{Change in } y}{\text{Change in } x} = \frac{\text{Rise}}{\text{Run}}$$

The slope of the line that contains the points $A(2, 3)$ and $B(0, -1)$ is:

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{-1-3}{0-2} = \frac{-4}{-2} = 2$$

73. USING AN EQUATION TO FIND THE SLOPE

To find the slope of a line from an equation, put the equation into the **slope-intercept** form:

$$y = mx + b$$

The slope is m. To find the slope of the equation $3x + 2y = 4$, re-express it:

$$\begin{aligned} 3x + 2y &= 4 \\ 2y &= -3x + 4 \\ y &= -\frac{3}{2}x + 2 \end{aligned}$$

The slope is $-\frac{3}{2}$.

standard form

$Ax + By = C$

point slope

$(y - y_1) = m(x - x_1)$

74. USING AN EQUATION TO FIND AN INTERCEPT

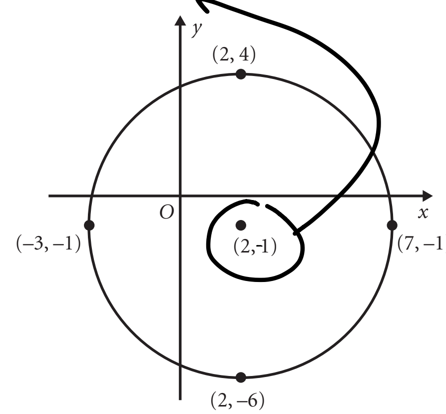
To find the y-intercept, you can either put the equation into $\boldsymbol{y = mx + b}$ (**slope-intercept**) form—in which case b is the y-intercept—or you can just plug $x = 0$ into the equation and solve for y. To find the x-intercept, plug $y = 0$ into the equation and solve for x.

75. EQUATION FOR A CIRCLE

The equation for a circle of radius r and centered at (h, k) is

$$(x - h)^2 + (y - k)^2 = r^2$$

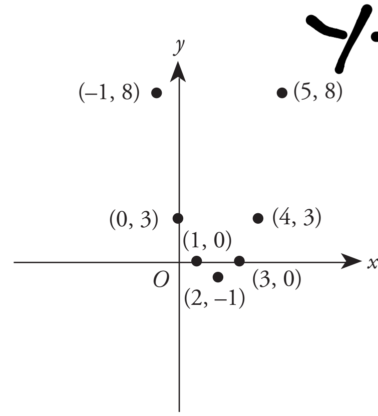
The figure below shows the graph of the equation $(x-2)^2 + (y+1)^2 = 25$:



76. EQUATION FOR A PARABOLA

The graph of an equation in the form $y = ax^2 + bx + c$ is a parabola. The figure below shows the graph of seven pairs of numbers that satisfy the equation $y = x^2 - 4x + 3$:

$y = a(x-h)^2 + k$



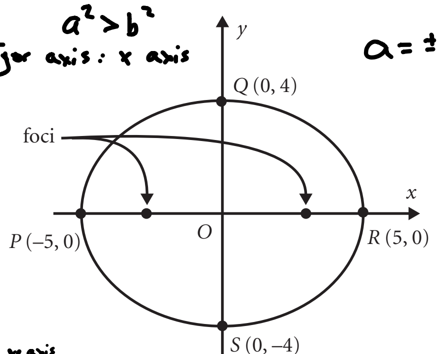
77. EQUATION FOR AN ELLIPSE

The graph of an equation in the form

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \qquad \frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

is an ellipse with $2a$ as the sum of the focal radii and with foci on the x-axis at $(0, -c)$ and $(0, c)$, where $c = \sqrt{a^2 - b^2}$. The figure below shows the graph of : $\frac{x^2}{25} + \frac{y^2}{16} = 1$ $\quad \sqrt{a^2} = \sqrt{25} \quad \sqrt{b^2} = \sqrt{16}$

$a^2 > b^2$ major axis: x axis $\quad a = \pm 5 \quad b = \pm 4$



$a^2 < b^2$ major axis: y axis

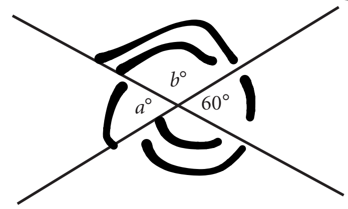
The foci are at (–3, 0) and (3, 0). *PR* is the **major axis**, and *QS* is the **minor axis**. This ellipse is symmetrical about both the *x*- and *y*-axes.

Lines and Angles

78. INTERSECTING LINES

When two lines intersect, **adjacent angles are supplementary** and **vertical angles are equal.**

|| 180°

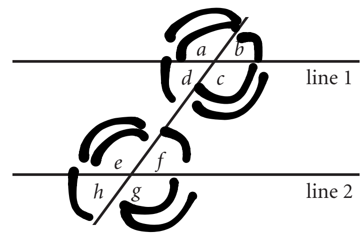


complimentary ∠'s = 90°

In the figure above, the angles marked $a°$ and $b°$ are adjacent and supplementary, so $a + b = 180$. Furthermore, the angles marked $a°$ and 60° are vertical and equal, so $a = 60$.

79. PARALLEL LINES AND TRANSVERSALS

A transversal across parallel lines forms **four equal acute angles and four equal obtuse angles**.



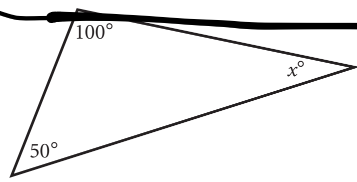
Here, line 1 is parallel to line 2. Angles *a*, *c*, *e*, and *g* are obtuse, so they are all equal. Angles *b*, *d*, *f*, and *h* are acute, so they are all equal.

Furthermore, **any of the acute angles is supplementary to any of the obtuse angles**. Angles *a* and *h* are supplementary, as are *b* and *e*, *c* and *f*, and so on.

Triangles—General

80. INTERIOR ANGLES OF A TRIANGLE

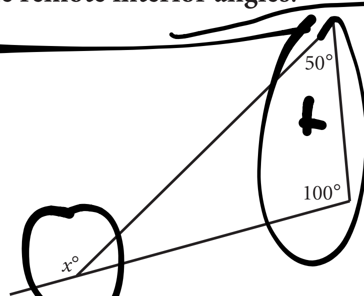
The three angles of any triangle **add up to 180°**.



In the figure above, $x + 50 + 100 = 180$, so $x = 30$.

81. EXTERIOR ANGLES OF A TRIANGLE

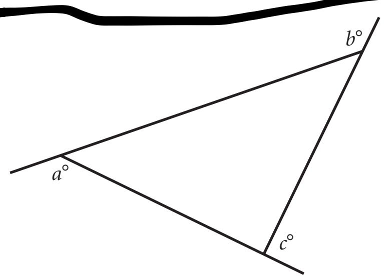
An exterior angle of a triangle is equal to the **sum of the remote interior angles**.



In the figure above, the exterior angle labeled $x°$ is equal to the sum of the remote interior angles:

$x = 50 + 100 = 150$.

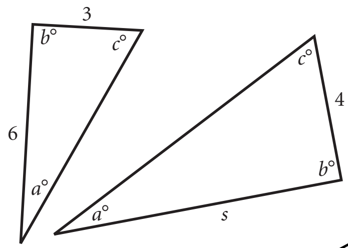
The three exterior angles of any triangle add up to 360º.



In the figure above, $a + b + c = 360$.

82. SIMILAR TRIANGLES

Similar triangles have the same shape: **correspon-ding angles are equal and corresponding sides are proportional**.



$\frac{4}{3} = \frac{s}{6}$

$\frac{24}{3} = \frac{3s}{3}$

$s = 8$

The triangles above are similar because they have the same angles. The 3 corresponds to the 4 and the 6 corresponds to the s.

$$\frac{3}{4} = \frac{6}{s}$$

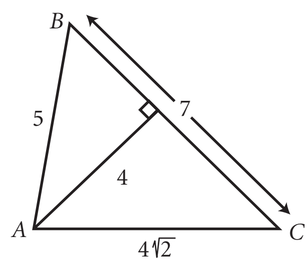
$$3s = 24$$

$$s = 8$$

83. AREA OF A TRIANGLE

$$\text{Area of Triangle} = \frac{1}{2}(\text{base})(\text{height})$$

The height is the perpendicular distance between the side that's chosen as the base and the opposite vertex.



In the triangle above, 4 is the height when the 7 is chosen as the base.

$$\text{Area} = \frac{1}{2}bh = \frac{1}{2}(7)(4) = 14$$

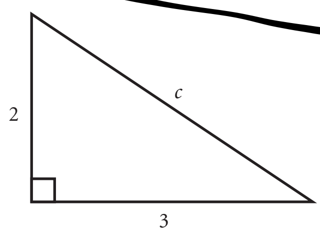
KNOW THESE

Right Triangles

84. PYTHAGOREAN THEOREM

For all right triangles:

$$(\text{leg}_1)^2 + (\text{leg}_2)^2 = (\text{hypotenuse})^2$$



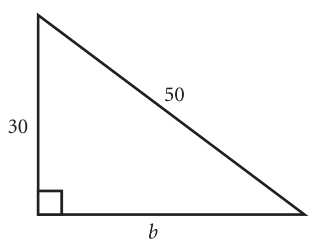
If one leg is 2 and the other leg is 3, then:

$$2^2 + 3^2 = c^2$$
$$c^2 = 4 + 9$$
$$c = \sqrt{13}$$

85. SPECIAL RIGHT TRIANGLES

- ***3-4-5***

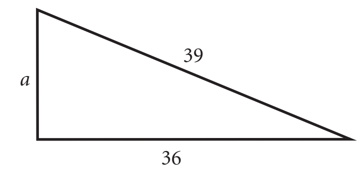
If a right triangle's leg-to-leg ratio is 3:4, or if the leg-to-hypotenuse ratio is 3:5 or 4:5, then it's a 3-4-5 triangle and you don't need to use the Pythagorean theorem to find the third side. Just figure out what multiple of 3-4-5 it is.



In the right triangle above, one leg is 30 and the hypotenuse is 50. This is 10 times 3-4-5. The other leg is 40.

- ***5-12-13***

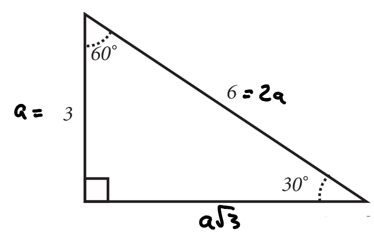
If a right triangle's leg-to-leg ratio is 5:12, or if the leg-to-hypotenuse ratio is 5:13 or 12:13, then it's a 5-12-13 triangle and you don't need to use the Pythagorean theorem to find the third side. Just figure out what multiple of 5-12-13 it is.



Here one leg is 36 and the hypotenuse is 39. This is 3 times 5-12-13. The other leg is 15.

- ***30-60-90***

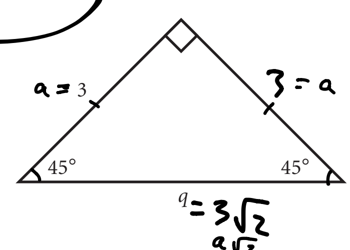
The sides of a 30-60-90 triangle are in a ratio of $\mathbf{1 : \sqrt{3} : 2}$. You don't need to use the Pythagorean theorem.



If the hypotenuse is 6, then the shorter leg is half that, or 3; and then the longer leg is equal to the short leg times $\sqrt{3}$, or $3\sqrt{3}$.

- ***45-45-90***

The sides of a 45-45-90 triangle are in a ratio of $\mathbf{1 : 1 : \sqrt{2}}$.



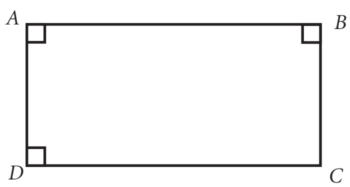
If one leg is 3, then the other leg is also 3, and the hypotenuse is equal to a leg times $\sqrt{2}$, or $3\sqrt{2}$.

Other Polygons

86. SPECIAL QUADRILATERALS

• ***Rectangle***

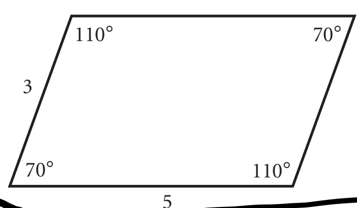
A rectangle is a **four-sided figure with four right angles**. Opposite sides are equal. Diagonals are equal.



Quadrilateral *ABCD* above is shown to have three right angles. The fourth angle therefore also measures 90°, and *ABCD* is a rectangle. The perimeter of a rectangle is equal to the sum of the lengths of the four sides, which is equivalent to 2(length + width).

• ***Parallelogram***

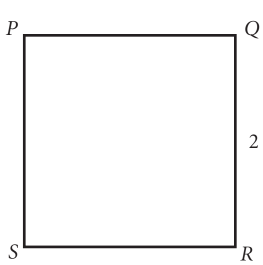
A parallelogram has **two pairs of parallel sides.** Opposite sides are equal. Opposite angles are equal. Consecutive angles add up to 180°.



In the figure above, *s* is the length of the side opposite the 3, so $s = 3$.

• ***Square***

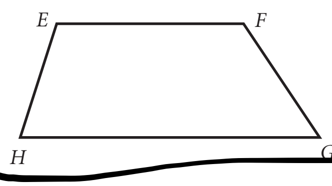
A square is a **rectangle with 4 equal sides**.



If *PQRS* is a square, all sides are the same length as *QR*. The perimeter of a square is equal to four times the length of one side.

• ***Trapezoid***

A **trapezoid** is a quadrilateral with one pair of parallel sides and one pair of nonparallel sides.

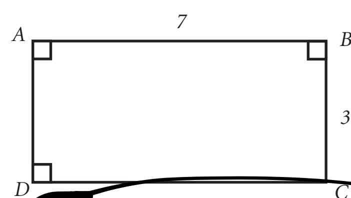


In the quadrilateral above, sides *EF* and *GH* are parallel, while sides *EH* and *GF* are not parallel. *EFGH* is therefore a trapezoid.

87. AREAS OF SPECIAL QUADRILATERALS

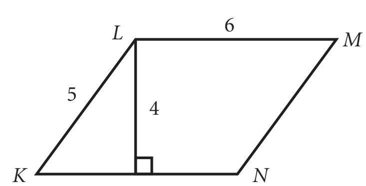
Area of Rectangle = Length × Width

The area of a 7-by-3 rectangle is $7 \times 3 = 21$.



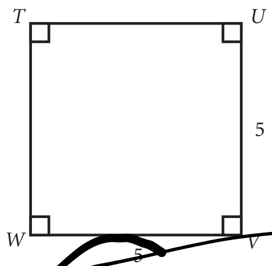
Area of Parallelogram = Base × Height

The area of a parallelogram with a height of 4 and a base of 6 is $4 \times 6 = 24$.



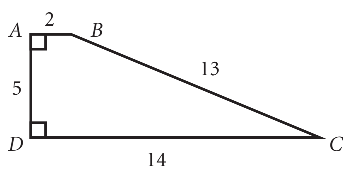
Area of Square = (Side)^2

The area of a square with sides of length 5 is 5^2 = 25.



Area of Trapezoid = $\left(\dfrac{\textbf{base}_1 + \textbf{base}_2}{\textbf{2}}\right) \times$ **height**

Think of it as the average of the bases (the two parallel sides) times the height (the length of the perpendicular altitude).



In the trapezoid *ABCD* above, you can use side *AD* for the height. The average of the bases is $\frac{2 + 14}{2} = 8$, so the area is 5×8, or 40.

88. INTERIOR ANGLES OF A POLYGON

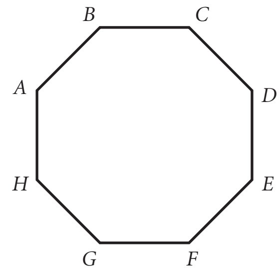
The **sum of the measures of the interior angles of a polygon is $(n - 2) \times 180$**, where n is the number of sides.

Sum of the angles = $(n - 2) \times 180$ degrees

The eight angles of an octagon, for example, add up to $(8 - 2) \times 180 = 1{,}080$.

To find **one angle of a regular polygon**, divide the sum of the angles by the number of angles (which is the same as the number of sides). The formula, therefore, is:

Interior angle = $\dfrac{(n - 2) \times 180}{n}$

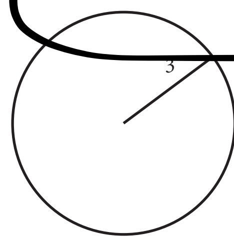


Angle *A* of the regular octagon above measures $\frac{1{,}080}{8}$ degrees.

Circles

89. CIRCUMFERENCE OF A CIRCLE

Circumference of a circle = $2\pi r$

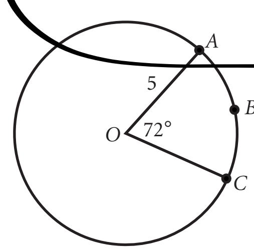


Here, the radius is 3, and so the circumference is $2\pi(3) = 6\pi$.

90. LENGTH OF AN ARC

An **arc** is a piece of the circumference. If n is the measure of the arc's central angle, then the formula is:

Length of an Arc = $\left(\dfrac{n}{360}\right)(2\pi r)$

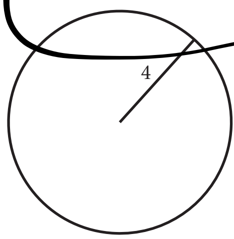


In the figure above, the radius is 5 and the measure of the central angle is 72°. The arc length is $\frac{72}{360}$ or $\frac{1}{5}$ of the circumference:

$$\left(\frac{72}{360}\right) 2\pi\, 5 = \left(\frac{1}{5}\right) 10\pi = 2\pi$$

91. AREA OF A CIRCLE

Area of a circle $= \pi r^2$

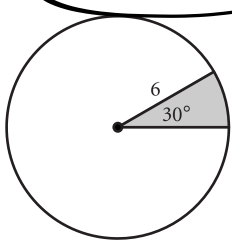


The area of the circle above is $\pi(4)^2 = 16\pi$.

92. AREA OF A SECTOR

A **sector** is a piece of the area of a circle. If n is the measure of the sector's central angle, then the formula is:

Area of a Sector $= \left(\frac{n}{360}\right)\left(\pi r^2\right)$



In the figure above, the radius is 6 and the measure of the sector's central angle is 30°. The sector has $\frac{30}{360}$ or $\frac{1}{12}$ of the area of the circle:

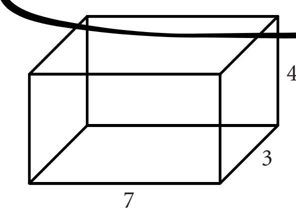
$$\left(\frac{30}{360}\right)(\pi)\left(6^2\right) = \left(\frac{1}{12}\right)(36\pi) = 3\pi$$

Solids

93. SURFACE AREA OF A RECTANGULAR SOLID

The surface of a rectangular solid consists of 3 pairs of identical faces. To find the surface area, find the area of each face and add them up. If the length is l, the width is w, and the height is h, the formula is:

Surface Area $= 2lw + 2wh + 2lh$

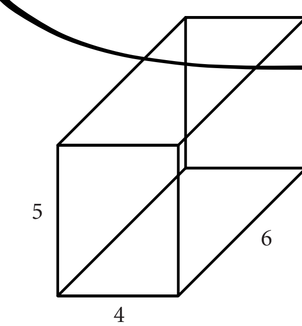


The surface area of the box above is:

$$\begin{aligned} &2 \times 7 \times 3 + 2 \times 3 \times 4 + 2 \times 7 \times 4 \\ &= 42 + 24 + 56 \\ &= 122 \end{aligned}$$

94. VOLUME OF A RECTANGULAR SOLID

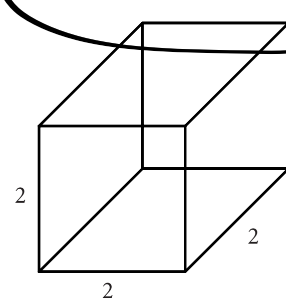
Volume of a Rectangular Solid $= lwh$



The volume of a 4-by-5-by-6 box is $4 \times 5 \times 6 = 120$

A cube is a rectangular solid with length, width, and height all equal. If e is the length of an edge of a cube, the volume formula is:

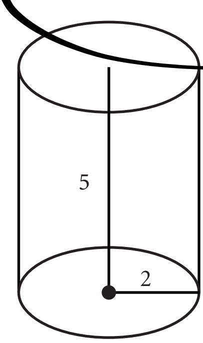
Volume of a Cube $= e^3$



The volume of the cube above is $2^3 = 8$.

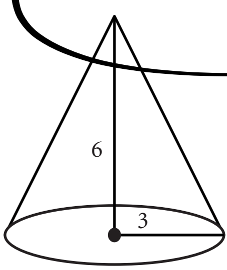
95. VOLUME OF OTHER SOLIDS

Volume of a Cylinder = $\pi r^2 h$



The volume of a cylinder where $r = 2$, and $h = 5$ is $\pi(2^2)(5) = 20\pi$

Volume of a Cone = $\frac{1}{3}\pi r^2 h$



The volume of a cone where $r = 3$, and $h = 6$ is:
Volume = $\frac{1}{3}\pi(3^2)(6) = 18$

Volume of a Sphere = $\frac{4}{3}\pi r^3$

If the radius of a sphere is 3, then:
Volume = $\frac{4}{3}\pi(3^3) = 36\pi$

Trigonometry

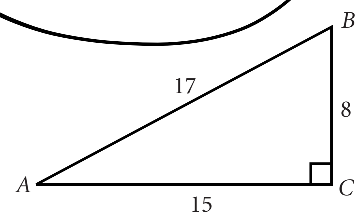
96. SINE, COSINE, AND TANGENT OF ACUTE ANGLES

To find the sine, cosine, or tangent of an acute angle, use **SOHCAHTOA,** which is an abbreviation for the following definitions:

$$Sine = \frac{\text{Opposite}}{\text{Hypotenuse}}$$

$$Cosine = \frac{\text{Adjacent}}{\text{Hypotenuse}}$$

$$Tangent = \frac{\text{Opposite}}{\text{Adjacent}}$$



In the figure above:

$\sin A = \frac{8}{17}$

$\cos A = \frac{15}{17}$

$\tan A = \frac{8}{15}$

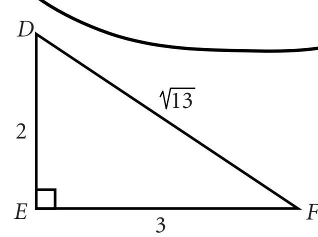
97. COTANGENT, SECANT, AND COSECANT OF ACUTE ANGLES

Think of the cotangent, secant, and cosecant as the reciprocals of the SOHCAHTOA functions:

$$Cotangent = \frac{1}{\text{Tangent}} = \frac{\text{Adjacent}}{\text{Opposite}}$$

$$Secant = \frac{1}{\text{Cosine}} = \frac{\text{Hypotenuse}}{\text{Adjacent}}$$

$$Cosecant = \frac{1}{\text{Sine}} = \frac{\text{Hypotenuse}}{\text{Opposite}}$$



In the figure above:

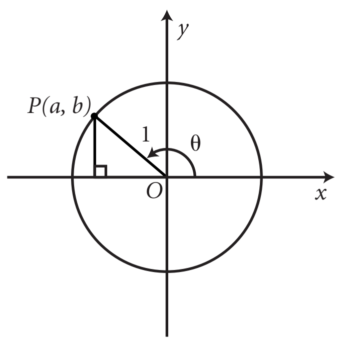
$\cot D = \frac{2}{3}$

$\sec D = \frac{\sqrt{13}}{2}$

$\csc D = \frac{\sqrt{13}}{3}$

98. TRIGONOMETRIC FUNCTIONS OF OTHER ANGLES

To find a trigonometric function of an angle greater than 90°, sketch a circle of radius 1 and centered at the origin of the coordinate grid. Start from the point (1, 0) and rotate the appropriate number of degrees counterclockwise.



In the "unit circle" setup on the previous page, the basic trigonometric functions are defined in terms of the coordinates a and b:

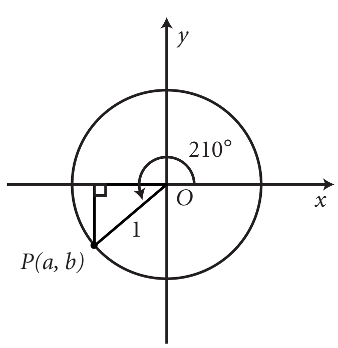
$\sin \theta = b$

$\cos \theta = a$

$\tan \theta = \frac{a}{b}$

Example: $\sin 210° = ?$

Setup: Sketch a 210° angle in the coordinate plane:



Because the triangle shown in the figure above is a 30-60-90 right triangle, we can determine that the coordinates of point P are $-\frac{\sqrt{3}}{2}, -\frac{1}{2}$. The sine is therefore $-\frac{1}{2}$.

99. SIMPLIFYING TRIGONOMETRIC EXPRESSIONS

To simplify trigonometric expressions, use the inverse function definitions along with the fundamental trigonometric identity:

$$\sin^2 x + \cos^2 x = 1$$

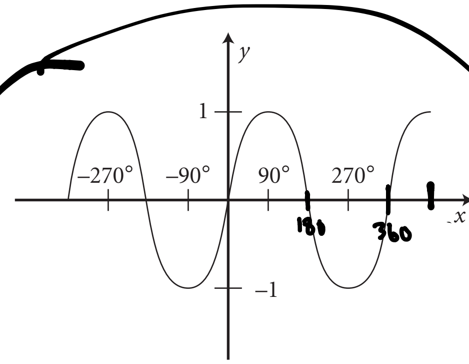
Example: $\frac{\sin^2 \theta + \cos^2 \theta}{\cos \theta} = ?$

Setup: The numerator equals 1, so:

$$\frac{\sin^2 \theta + \cos^2}{\cos \theta} = \frac{1}{\cos \theta} = \sec \theta$$

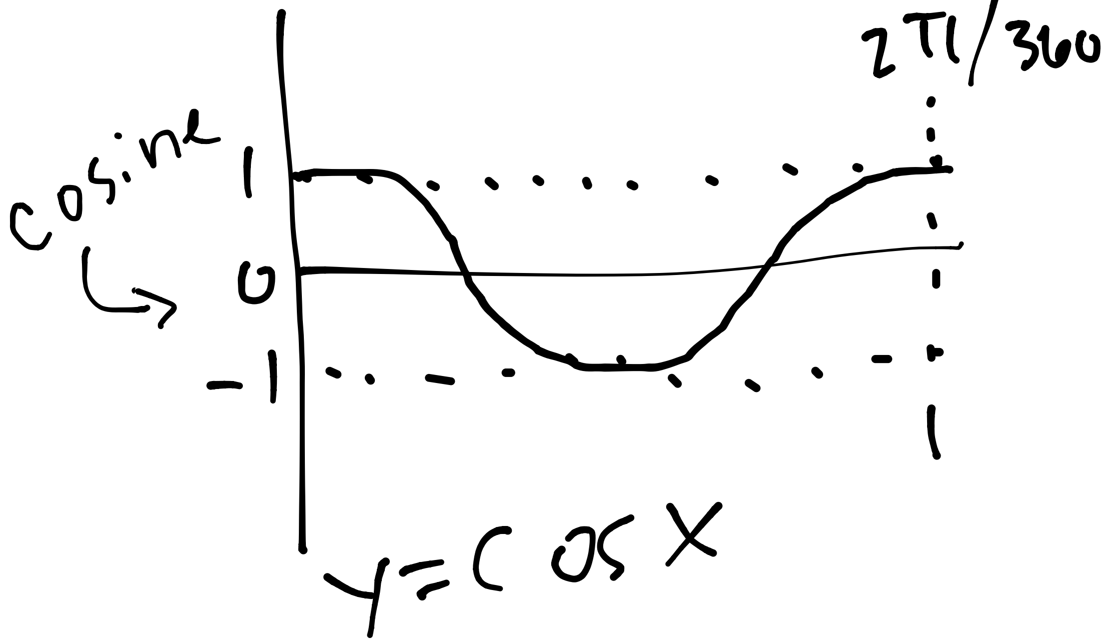
100. GRAPHING TRIGONOMETRIC FUNCTIONS

To graph trigonometric functions, use the x-axis for the angle and the y-axis for the value of the trigonometric function. Use special angles—0°, 30°, 45°, 60°, 90°, 120°, 135°, 150°, 180°, etc.—to plot key points.



The figure above shows a portion of the graph of $y = \sin x$.

Sine



arithmatic sequence

1st #

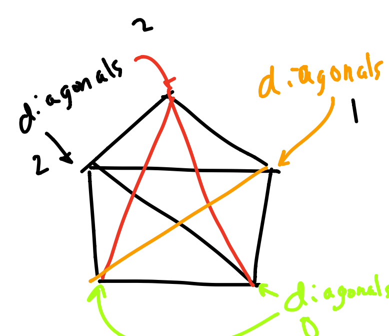
$$a_n = a_1 + d(n-1)$$

nth term — difference — nth term

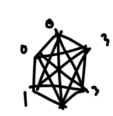
geometric sequence

$$g_n = g_1 (r)^{n-1}$$

nth term — ratio — 1st number



$$\boxed{\frac{n(n-3)}{2} = \#\text{ diagonals}}$$

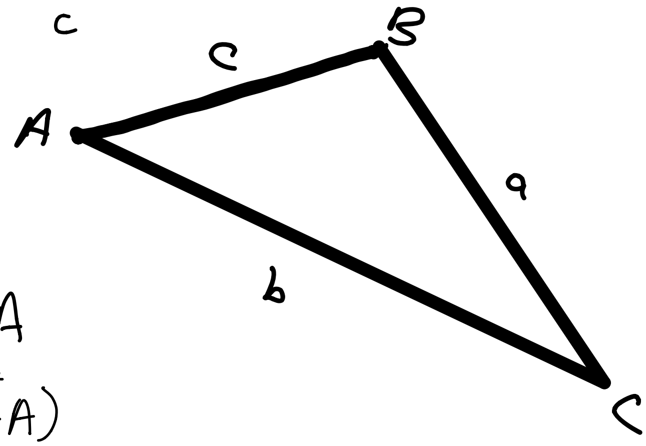


$$\frac{6(6-3)}{2} = 9$$

midpoint formula: $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

Law of sines

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$



Law of cosines

$$a^2 = b^2 + c^2 - 2ab\cos A$$

example

SAS$(b, c, \angle A)$